

Requirement of a Primordial Magnetic Field in Chameleon Models

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We show that the presence of primordial magnetic fields (PMFs) are imperative in order to satisfy constraints placed on chameleon models from variation of particle masses at big bang nucleosynthesis. For initial field values after inflation of order M_{Pl} , the combined magnetic field strength B_0 and chameleon-photon coupling $1/M_F$ must satisfy $(B_0/5\text{nG})^2 (M_F/10^9\text{GeV})^{-1} \gtrsim 2.4 \times 10^{-6}$. Combining these constraints with those derived from considering the degree of mixing between chameleons and CMB photons in a PMF, implies the existence of a primordial magnetic field with strength $B_0 \gtrsim 0.1\text{nG}$, for theories of Modified Gravity with a chameleon mechanism to be viable.

Gravity theories which extend General Relativity by introducing a new degree of freedom have received increased attention lately due to combined motivation coming from high-energy physics, cosmology and astrophysics [1]. If they claim to account for dark energy or dark matter, then they are only valid if they pass the many weak-field limit tests. For many this is only possible if they have a chameleon mechanism [1, 2]. Such mechanisms have the effective mass of the scalar degree of freedom being a function of the curvature (or energy density) of the local environment, so that in effect the mass is large at Solar System and terrestrial curvatures and densities, and small at cosmological curvatures and densities. It is short-ranged in the Solar System, but becomes long-ranged at cosmological densities and can drive the acceleration of the universe.

In the Einstein frame, the new degree of freedom can be identified with the chameleon field. The early universe behaviour of the chameleon, considering only the interaction of the chameleon to matter species, was investigated in [3]. Assuming that the chameleon is produced at some phase transition during inflation, and that at the end of inflation the chameleon is left at some arbitrary field value, it then rolls to the minimum of its potential and in some cases oscillates about it. Variation of particle masses after big bang nucleosynthesis (BBN) requires the maximum amplitude of the chameleon oscillations to be below a certain threshold at BBN. Satisfying these constraints is of utmost importance in building a viable theory of Modified Gravity. In [3] the initial conditions of the chameleon field needed to be strongly fine-tuned at the end of inflation to satisfy the BBN constraints. In this *Letter* we show that in general the presence of a large scale PMF in the early universe is required to drive the chameleon towards the minimum in time for BBN.

The existence of a large scale magnetic field in the early universe is a matter of current debate. The WMAP 7-year results place a maximum bound on its magnitude of $B_0 < 5.0\text{nG}$ [8]. Although the presence of a PMF has not been detected, it is generally believed to be required for the formation of the order μG magnetic fields observed in galaxies and galaxy clusters. See [13] for a recent review.

In the chameleon model the matter particles couple indirectly to the chameleon ϕ via a conformal metric $B_m^2(\phi)g_{\mu\nu}$. The coupling of the chameleon to charged matter naturally generates a direct coupling between the chameleon and electromagnetic field of the form $B_F(\phi)F_{\mu\nu}F^{\mu\nu}$ [4]. We take the coupling functions to be of generic exponential form: $B_m(\phi) \simeq \exp(\phi/M)$ and $B_F(\phi) \simeq \exp(\phi/M_F)$. The strongest bounds on the matter coupling come from particle physics experiments, $M \gtrsim 10^4\text{GeV}$ [5]. Stronger constraints have been derived on the chameleon-photon coupling strength. Constraints on the production of starlight polarization in the galactic magnetic field place a lower bound of $M_F \gtrsim 1.1 \times 10^9\text{GeV}$ [6], while measurements of the Sunyaev-Zel'dovich effect in galaxy clusters places a lower bound in the range $M_F \gtrsim (0.25-1.14) \times 10^9\text{GeV}$, depending on the model assumed for the cluster magnetic field [7].

The equation describing the chameleon evolution in the presence of a magnetic field is,

$$-\phi_{,tt} - 3H\phi_{,t} = V'(\phi) + \frac{1}{2}|B|^2 B_F'(\phi) - T_\mu^\mu B_m'(\phi), \quad (1)$$

where we have assumed a spatially flat FRW metric. The universe becomes a good conductor soon after the end of inflation, and from then on all magnetic fields are frozen-in with $|B|^2 \propto a^{-4}$. The evolution of the scale factor $a(t)$ is described by the Hubble expansion, $H \equiv \dot{a}/a$. At these early times the universe is radiation dominated and we can approximate $H^2(a) \simeq H_0^2 \Omega_{r0} a^{-4}$, where $\Omega_{r0} \equiv 8\pi G \rho_{r0}/3H_0^2$ is the fractional energy density of radiation.

In general the stress-energy tensor $T_\mu^\mu = -\rho_m$, where ρ_m is the energy density in non-relativistic matter. However as the universe cools and the particle species drop out of thermal equilibrium, there is an additional short-lived boost to T_μ^μ which has a significant effect on the chameleon evolution. For the chameleon mechanism we require the self-interaction potential of the scalar field to be of runaway form. A typical choice is, $V(\phi) = \Lambda^4 \exp(\Lambda^n/\phi^n)$, where $n \sim \mathcal{O}(1)$ and we require $\Lambda \simeq 2.4 \times 10^{-3}\text{eV}$ for a suitable dark energy candidate.

Equation (1) is that of a weakly damped oscillator, moving in an effective potential defined by,

$$V_{\text{eff}}(\phi) = \Lambda^4 \exp\left(\frac{\Lambda}{\phi}\right)^n + \frac{1}{2}|B|^2 \exp\left(\frac{\phi}{M_F}\right) + \rho_m \exp\left(\frac{\phi}{M}\right),$$

where for now we neglect the extra contribution to T_μ^μ as particle species go non-relativistic. When in equilibrium the chameleon seeks the minimum of this effective potential, ϕ_{\min} . Notice that $M, M_F \gg \Lambda$ and so the lefthand side of the potential with $\phi < \phi_{\min}$ is many orders of magnitude steeper than that for $\phi > \phi_{\min}$. The exponential decay of the functions means that for $\phi < \phi_{\min}$ the potential is almost entirely dominated by $V(\phi)$, while for $\phi > \phi_{\min}$ the dominant driving force is from the ρ_m and $|B|^2$ terms. An exact solution to the differential equation (1) cannot be found but we use the following approximation to determine the evolution of the chameleon. Whenever $\phi > \phi_{\min}$ we assume $V(\phi)$ is negligible. Approximating $B_m(\phi), B_F(\phi) \approx 1$, we can solve for the evolution along this side of the potential:

$$\begin{aligned} \frac{d\phi}{da} &= -\beta a^{-1} + Aa^{-2}, \\ \phi(a) &= -\beta \log a - Aa^{-1} + B, \end{aligned} \quad (2)$$

where $\beta \equiv |B|^2/2M_F\Omega_{r0}H_0^2$. We neglect the contribution from ρ_m since this is subdominant prior to BBN. The constants A and B are determined by the initial conditions. Conversely, when $\phi < \phi_{\min}$ the $|B|^2$ term is negligible compared to $V(\phi)$. The friction term proportional to H in equation (1) is also negligible since the steepness of the potential dominates. The roll up or down on this side of the potential is very rapid and we treat it as instantaneous. As the chameleon approaches from larger field values, $V(\phi)$ effectively acts as a perfect elastic collision.

We assume the chameleon starts from rest at some initial value ϕ_i at the end of inflation. For starting values with $\phi_i \lesssim \phi_{\min}$, the chameleon falls very quickly to the minimum and overshoots to the other side of the potential before coming to a halt. It then starts to fall back towards the minimum and evolves identically to the case with $\phi_i > \phi_{\min}$. Depending on the size of the background magnetic field and the initial starting value for the chameleon field, the chameleon either falls to the minimum and start oscillating about it, or the friction term dominates and the field remains frozen at its initial value. The amplitude of the oscillations gradually decay due to the damping term from the Hubble expansion.

Consider a single oscillation which starts from rest at $\phi = \phi_1$ at some time a_1 with $\phi_1 \gg \phi_{\min}$. The field is governed by equation (2) as it rolls to the minimum. We then assume at ϕ_{\min} that it undergoes an instantaneous perfect elastic collision and rebounds with the same velocity. It comes to a halt at the retracement point ϕ_2 at some time a_2 :

$$a_2 \simeq 2a_{\min} - a_1 \quad (3)$$

$$\phi_1 - \phi_2 \simeq \beta \left[\log \left(\frac{a_2}{a_1} \right) + 2 \left(\frac{a_1}{a_{\min}} - 1 \right) \right], \quad (4)$$

where the scale factor when the field reaches the mini-

mum a_{\min} is determined by,

$$\log \left(\frac{a_{\min}}{a_1} \right) + \frac{a_1}{a_{\min}} \simeq \frac{\phi_1}{\beta} + 1. \quad (5)$$

This evolution from one retracement point (ϕ_1, a_1) to another (ϕ_2, a_2) can be iterated to determine the chameleon behaviour. An analytic approximation for the evolution of the retracement point ϕ_{\max} exists in the limit of fast oscillations when $a_{\min}/a_1 \approx 1 + \delta$, $\delta \ll 1$:

$$\log \phi_{\max} \simeq -\frac{2}{3} \log a + \text{const.} \quad (6)$$

This power law behaviour is independent of the strength of the magnetic field. In this regime, the magnetic field strength dictates how rapidly the chameleon oscillates but not how long it takes to settle to the minimum.

In addition to these oscillations there is the extra contribution to T_μ^μ as particle species drop out of thermal equilibrium. This is a short-lived boost to the driving term on the righthand side of equation (1). We follow the convention in [3] and refer to these contributions as ‘kicks’. The first ‘kick’ occurs at a redshift of approximately $z \sim 10^{14}$ as the top quark drops out of thermal equilibrium. The contribution as each particle, labelled by k , goes non-relativistic is [3, 11],

$$T_\mu^{\mu(k)} = -\frac{45}{\pi^4} H^2(a) M_{\text{Pl}}^2 \frac{g_k}{g_*(T)} \tau \left(\frac{m_k}{T} \right),$$

where

$$\tau(x) \equiv x^2 \int_x^\infty du \frac{\sqrt{u^2 - x^2}}{e^u \pm 1},$$

and the \pm sign is for fermions and bosons respectively. The τ function is approximately $\mathcal{O}(1)$ for $x \sim 1$ and negligible otherwise, and so the maximum contribution occurs when the temperature of the universe has cooled sufficiently for it to match the mass of the particle, $m_k \sim T$. The mass of the different species, along with the ratio of the number of degrees of freedom to the effective number of relativistic degrees of freedom, g_k/g_* , are in Table I.

An exact solution to the chameleon evolution with the ‘kicks’ is not possible. Instead we consider two limiting cases: when the duration of the ‘kick’ is very much less than the oscillation period of the chameleon, we can approximate the contribution to T_μ^μ as a delta function; on the other hand when the chameleon is oscillating rapidly about the minimum, we can make an adiabatic approximation. The duration of the ‘kick’ can be approximated as $a_2/a_1 \approx 100$, where $a_2 - a_1 = \Delta a_{\text{kick}}$, for which the τ function has dropped to less than 10% of its maximum value.

Approximating $\tau(x)$ as a delta-function centered on $a = a_k$, we have for each ‘kick’,

$$T_\mu^{\mu(k)} \approx -\frac{45}{\pi^4} H^2(a) M_{\text{Pl}}^2 \frac{g_k}{g_*(a_k)} a_k \Gamma \delta(a - a_k),$$

Particle	mass, m_k	g_k/g_* (at $T = m_k$)	type
t	173 GeV	12/106.75	fermion
Z	91 GeV	3/95.25	boson
W^\pm	80 GeV	6/92.25	boson
b	4 GeV	12/86.25	fermion
τ	1.7 GeV	4/75.75	fermion
c	1.27 GeV	12/72.25	fermion
π	0.14 GeV	3/17.25	boson
μ	0.105 GeV	4/14.25	fermion
e	0.5 MeV	4/10.75	fermion

TABLE I: List of particle species that provide a ‘kick’ to the chameleon evolution as they drop out of thermal equilibrium.

where $a_k \equiv T_0/m_k$, and $\Gamma \approx 4.9\text{--}4.6$ is the total area under the $\tau(x)$ curve. We find that the jumps in the chameleon field and its velocity from their free evolution as a result of the kick are,

$$\begin{aligned}\Delta\phi_{,a} &\simeq -\kappa_k M_{\text{Pl}} \Gamma a_k a^{-2}, \\ \Delta\phi &\simeq -\kappa_k M_{\text{Pl}} \Gamma \left(1 - \frac{a_k}{a}\right),\end{aligned}$$

where $\kappa_k \equiv 45M_{\text{Pl}}(g_k/g_*)/\pi^4 M$.

When the duration of the ‘kick’ is significantly greater than the oscillation period of the chameleon, we use an adiabatic approximation. Treating $\tau(a/a_k)$ as constant over one oscillation, we find that the evolution is identical to equation (2) but with β replaced by,

$$\beta_{\text{eff}} \equiv \beta + M_{\text{Pl}} \sum_k \kappa_k \tau \left(\frac{a_1}{a_k}\right),$$

where a_1 is the scale factor at the start of the oscillation. In this regime the effect of the kicks is similar to a temporary increase in the magnetic field strength, and generally boosts the oscillations into the ‘fast oscillations’ regime described by equation (6).

In Figure 1 we plot two examples of the predicted chameleon evolution from the end of inflation ($z \sim 10^{23}$) until BBN ($z \sim 10^8$) for different parameter values. Note that the evolution depends on β and as such is degenerate between B_0 and M_F . The parameter,

$$\lambda \equiv \left(\frac{B_0}{5\text{nG}}\right)^2 \left(\frac{M_F}{1.1 \times 10^9 \text{GeV}}\right)^{-1},$$

characterizes this dependence.

Nucleosynthesis constrains the variation in particle masses from BBN until the present day to be less than approximately 10% [12]. The particle masses in the chameleon model are dependent on ϕ with $m(\phi) = B_m(\phi)m_0$ [9], and so variation in ϕ is constrained to be less than $0.1M$. Since $\phi_{\text{min}}^{(\text{today})} \approx 10^8 \Lambda \ll 0.1M$, this leads to $\phi^{(\text{BBN})} \lesssim 0.1M$.

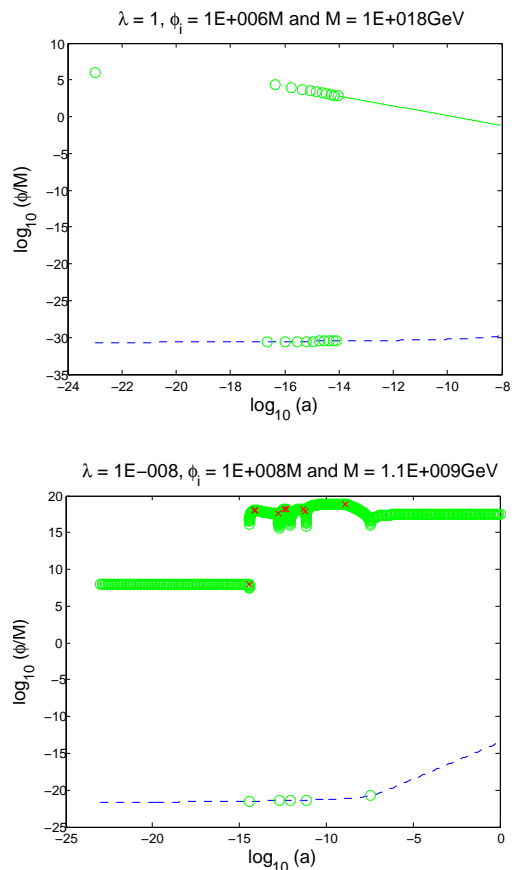


FIG. 1: Chameleon evolution for different λ , M and ϕ_i . The dashed blue line is the evolution of ϕ_{min} . Green circles mark the actual location of the field including the oscillations, while the green line is the evolution of ϕ_{max} in the ‘fast oscillations’ regime. Red crosses mark the location of the ‘kicks’.

We determine the parameter sets that satisfy these BBN constraints and plot the exclusion bounds in Figure 2 for two choices of matter coupling strength: $M = 1.1 \times 10^9 \text{GeV}$ and $M = M_{\text{Pl}}$. The ratio of ϕ_i to λ dictates whether the chameleon has started to oscillate prior to the top quark dropping out of thermal equilibrium (upper plot of Figure 1) in which case it quickly enters the ‘fast oscillations’ regime with the extra boost from the kicks; or whether the field is frozen at its initial value until the first kick occurs and sends it towards the minimum. We find that in all cases when the field is frozen at its initial value, the BBN constraints are not satisfied. The rebound at the minimum of the potential often causes the chameleon to shoot to even larger field values by the end of the ‘kicks’ (lower plot in Figure 1). The field is frozen at its initial value for all cases when the interaction with the PMF is neglected, and it requires extreme fine tuning of the initial conditions to satisfy the BBN constraints.

The transition between an oscillating chameleon and a frozen chameleon prior to the kicks occurs when the field

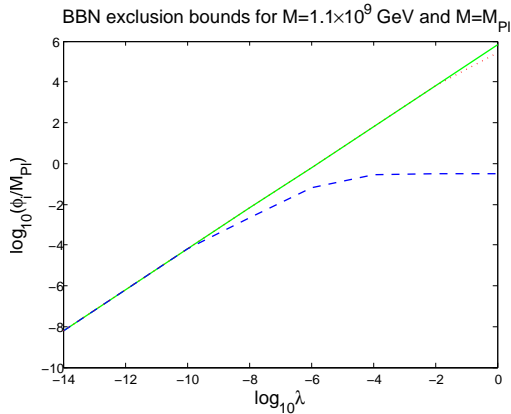


FIG. 2: The parameter space of ϕ_i and λ that is excluded by BBN constraints. The dashed blue line marks the excluded region for $M = 1.1 \times 10^9$ GeV, and the dotted red for $M = M_{\text{Pl}}$. The green line marks the transition from the chameleon oscillating prior to the ‘kicks’, and the one in which the field is frozen prior to the kicks.

has oscillated exactly once between $a \sim 10^{-23}$ and the first kick at $a = 3.8 \times 10^{-15}$. Under the approximation $a_{\text{min}} \gg a_1$, we find this transition occurs when,

$$\phi_i \approx 3.8 \times 10^5 M_{\text{Pl}} \lambda.$$

The transition between when the chameleon is oscillating and when it is frozen at its initial value also marks the BBN exclusion bound for small ϕ_i but, as we can see from Figure 2, there is a saturation point dependent on M but independent of λ which excludes larger values of ϕ_i . This occurs when the chameleon is in the ‘fast oscillations’ regime described by equation (6) for all of its evolution and so we have,

$$\phi_{\text{max}}^{(\text{BBN})} \approx 10^{-10} \phi_i.$$

To satisfy the BBN constraints this imposes, $\phi_i \lesssim 10^9 M$. Approximating these bounds as straight lines, we obtain the following constraints on M , M_{F} and B_0 for a given ϕ_i :

$$M \gtrsim 10^{-9} \phi_i, \quad \text{and} \quad B_{5\text{nG}}^2 M_9^{-1} \gtrsim 2.4 \times 10^{-6} \frac{\phi_i}{M_{\text{Pl}}},$$

where $B_{5\text{nG}} \equiv B_0/5\text{nG}$ and $M_9 \equiv M_{\text{F}}/10^9\text{GeV}$. For example when $\phi_i \sim \mathcal{O}(M_{\text{Pl}})$ we require,

$$M \gtrsim 10^9 \text{GeV}, \quad \text{and} \quad B_{5\text{nG}}^2 M_9^{-1} \gtrsim 2.4 \times 10^{-6}.$$

In this *Letter* we have demonstrated that it is essential to have a large scale magnetic field in the early universe for the chameleon to satisfy bounds placed on variation of particle masses at BBN. The results of [3] that did not require a PMF are strongly fine tuned to specific initial conditions for the chameleon at the end of inflation.

For initial field values up to order M_{Pl} , we find that the chameleon to matter coupling strength is constrained at $M \gtrsim 10^9 \text{GeV}$, which is many orders of magnitude greater than the existing direct constraints on M . In addition there is a degenerate constraint on the PMF strength and chameleon-photon coupling, $B_{5\text{nG}}^2 M_9^{-1} \gtrsim 2.4 \times 10^{-6}$. This can be compared to the constraints placed on B_0 and M_{F} from considering CMB photons mixing with chameleons in a PMF [10]: $B_{5\text{nG}} M_9^{-2} \lesssim 6 \times 10^{-7}$ at 95% confidence (for a PMF spectral index of $n_B = -2.9$).

The allowed degree of chameleon-photon mixing in the early universe places an upper bound on the strength of the interaction between the chameleon and a PMF, while for the chameleon to be well behaved in the early universe and satisfy constraints at BBN we require a strong magnetic interaction to drive the chameleon to its minimum in time. Combining the two bounds, we find there is only a narrow band of parameter values that satisfy these constraints,

$$1.3 \times 10^4 B_{5\text{nG}}^{\frac{1}{2}} \lesssim M_9 \lesssim 4 \times 10^5 B_{5\text{nG}}^2.$$

For primordial magnetic fields smaller than 10^{-10}G , there are no coupling strengths that can satisfy the combined bound, and so in all cases we require $B_0 \gtrsim 10^{-10} \text{G}$. This result opens up an altogether new window which might lead to a completely different view of the role played by early universe magnetic fields when building viable extensions to General Relativity.

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